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An Algorithm to Calculate Phase-Center Offset of Aperture Antennas when Measuring 2-Dimensional Radiation Patterns

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14. ABSTRACT

Centering the phase center of an antenna onto the rotational axis used to measure its radiation pattern is an iterative and time-consuming process. To facilitate this, a MATLAB algorithm has been developed to calculate the phase-center offset from the axis of rotation of a 2-dimensional antenna pattern. The hybrid algorithm comprises a combination of the 2-point method to calculate the offset along the antenna's main beam and an antisymmetry method is used to calculate offset perpendicular to the main-beam direction. The MATLAB algorithm is tested on the E-plane radiation pattern of a cylindrical horn antenna calculated using the high-frequency structural simulator's electromagnetic simulation engine, radiating at 5 GHz. The algorithm calculates the phase-center offset to within 15%. Because the algorithm analyzes the unwrapped phase of the radiation pattern, which it converts to offset distance, no ambiguity due to offsets greater than a wavelength exists. Using this algorithm, the phase center of the antenna can be placed coincident to the axis of rotation after the first antenna pattern is measured and analyzed.

15. SUBJECT TERMS

Phase-center, radiation-pattern measurement, phase-center error

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1. Introduction

Measuring the radiation pattern of a directive antenna when its phase center is not aligned with the axis of rotation yields large far-field radiation phase changes. If severe enough, such misalignment can have a significant effect on the amplitude symmetry of the radiation pattern about its main-beam direction. Normally, the antenna placement is iterated until the phase-center is colocated on the axis of rotation. This trial-and-error placement can be time consuming as many (partial) radiation patterns must be measured and the data analyzed to find the antenna placement with the least phase variation in the antenna pattern.

A need has been identified to calculate the phase-center offset from the axis of rotation of 2-dimensional radiation-pattern data after the first radiation pattern has been measured. With the phase-center offset magnitude and direction known, the antenna can be correctly adjusted so that its phase-center location becomes coincident with the axis of antenna rotation.

Methods of calculating the location of the phase center on an aperture antenna have been published.[1-3] These techniques concern themselves mostly with finding the antenna's phase center when not coincident with the antenna's aperture face and thus calculate the phase-center offset in the main-beam (boresight) direction. Because antenna-placement error can offset the phase center in both the main-beam and lateral directions, not only the offset magnitude but also the offset direction must be calculated. The algorithm reported here is a geometric analysis of the complex radiation-pattern phase. The radiation phase is separated into its symmetric and antisymmetric components, and different methods are used to solve for the components of phase offset. The main-beam offset component is solved from the symmetric component of the radiation phase using the law of cosines; the lateral component of the phase-center offset is solved using the antisymmetric component of the radiation phase using a simple phase-excursion method.

The azimuthal difference in phase is translated into relative distance from the main-beam phase's center distance to obtain the phase-center offset in the main-beam direction, and a simple phase-excursion method is used to obtain the phase-center offset laterally to the main beam's direction. The results of the 2 methods are combined to yield the phase-center offset in both magnitude and direction, with no ambiguity due to the offset being greater than a wavelength.

2. Methodology

The 2-point method[1] assumes a phase difference (due to the phase-center offset from the rotational axis) that is symmetric about the apex of the main beam, while the second method

assumes that the phase difference is antisymmetric about the apex of the main beam. By separating the radiation pattern's phase into its symmetric and antisymmetric components about the antenna's main beam, the phase data can be used to calculate the offset in both the main-beam and lateral directions. The 2 offset components are combined to yield the direction and magnitude of the phase-center relative to the axis of rotation of the pattern-measurement equipment. Figure 1 shows the magnitude a) and phase b) of the cylindrical horn antenna's radiation pattern offset in both the main-beam and lateral directions by 1 cm. The phase of the radiation pattern clearly shows the effect of the lateral offset.
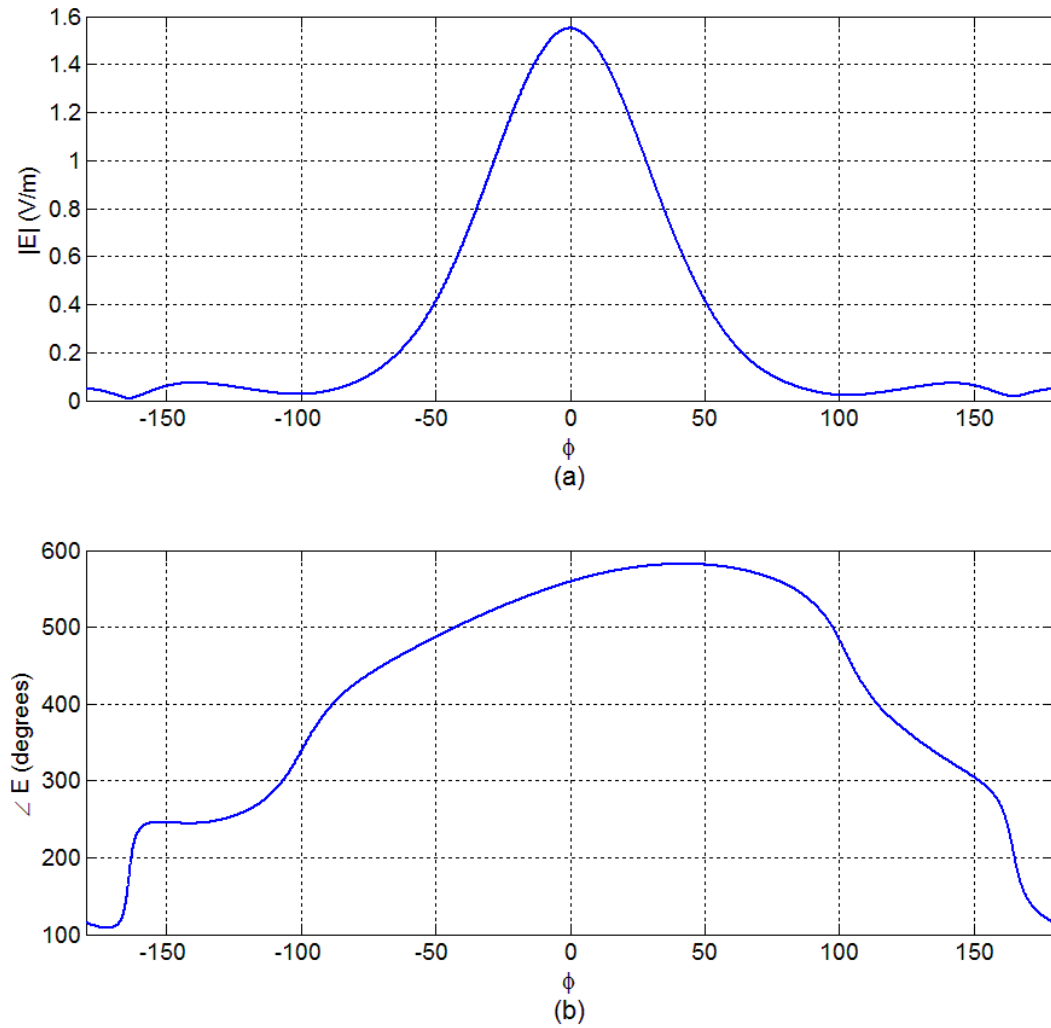

Fig. 1   Magnitude a) and phase b) of the radiation pattern of the cylindrical antenna offset 1 cm in both the main-beam and lateral directions

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2.1 Separation of the Symmetric and Antisymmetric Components of the Radiation-Pattern Phase

To separate the symmetric (even) and antisymmetric (odd) components of the data, the antenna's main beam is shifted to coincide with the middle point of the data set. Then the even and odd components are found using the general formulas

$$f(n)_{even} = \frac{f(n) + f(-n)}{2}$$
$$f(n)_{odd} = \frac{f(n) - f(-n)}{2} \qquad , \tag{1}$$

where $f(-n)$ refers to the flipped data, where the last data point is set as the first.

Figure 2 shows the symmetric (a) and antisymmetric (b) components of the phase of the cylindrical horn antenna offset 1 cm in the main-beam and lateral directions.
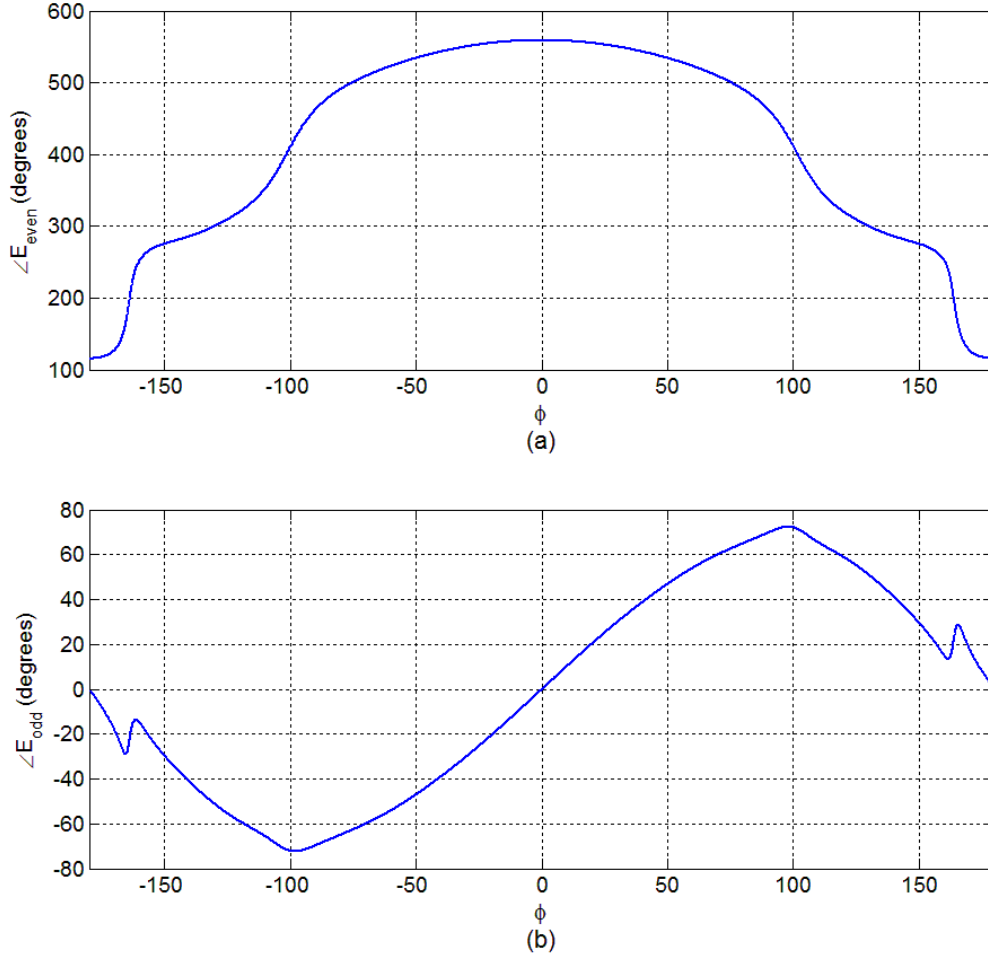

Fig. 2 Symmetric a) and antisymmetric b) components of the radiation-pattern phase of the cylindrical antenna offset 1 cm in the main-beam and lateral directions

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2.2 Phase-Center Offset in the Direction of the Main Beam

The 2-point method[1] assumes that the phase of the radiation pattern of an aperture antenna should not have azimuth-angular variation if the phase center is coincident with the rotational axis of the antenna. Thus the difference in phase from the main-beam direction to some azimuthal offset (±90° in this work) of the radiation pattern indicates the electrical distance between the phase center and the axis of rotation in the main-beam direction.

Figure 3 shows the symmetric component of the phase of the radiation pattern of a cylindrical horn antenna that has its phase-center offset 1 cm in the main-beam direction.

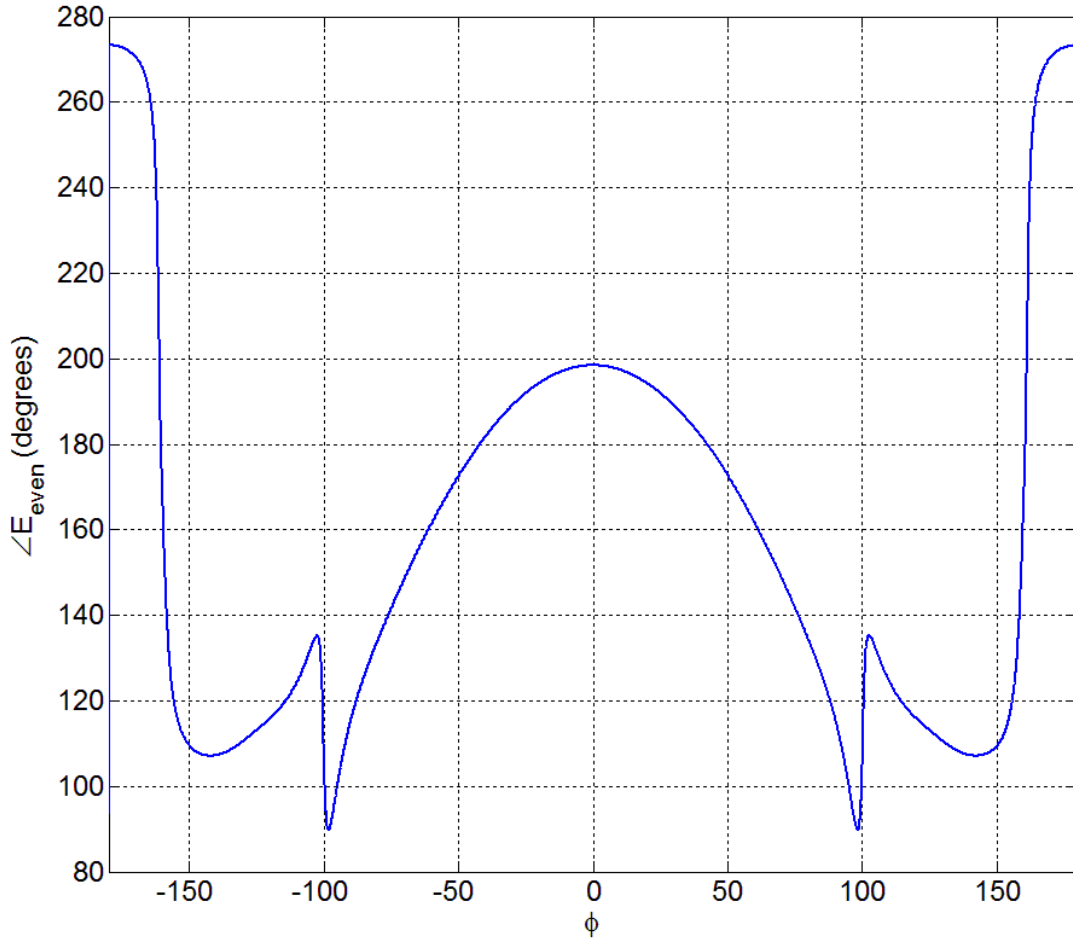Fig. 3   Radiation-pattern phase of a cylindrical horn antenna whose phase center is offset 1 cm from the axis of rotation in the main-beam direction

The figure shows that the phase of the antenna's radiation pattern changes significantly at ±20° from the value along boresight. This change in radiation-pattern phase as the antenna is rotated in azimuth is used to calculate the phase-center offset from the antenna's rotational axis.

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Figure 4 shows the geometry used to calculate the component of the phase-center offset in the direction of the main beam. Note that, unlike published methods,[1–3] the phase-center offset can contain a lateral component in this geometry.
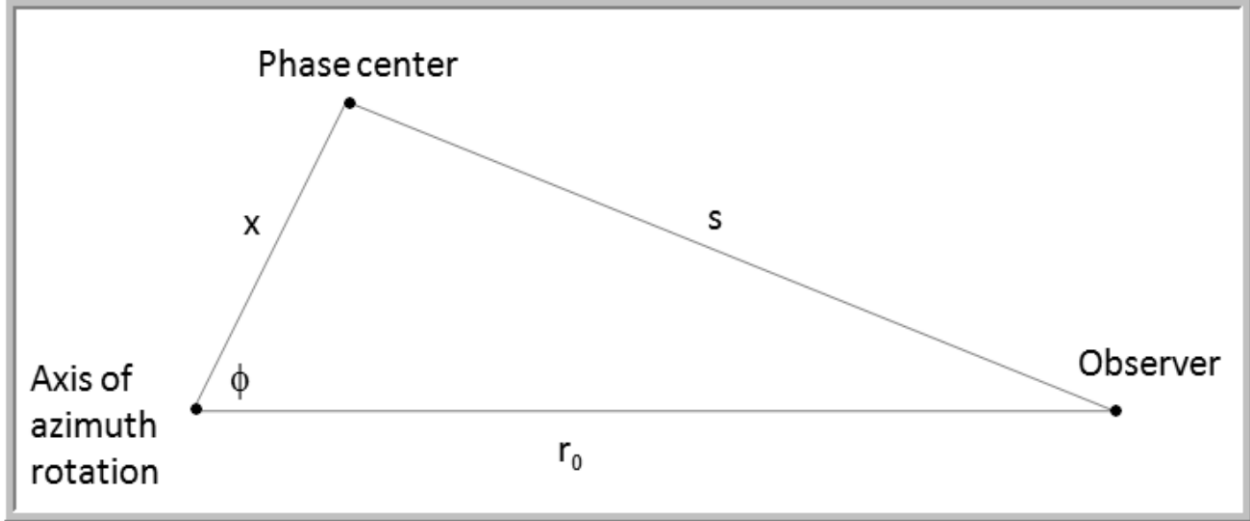

Fig. 4 Geometry used to calculate the main-beam direction component of the phase-center offset

In Fig. 4, $\emptyset$ is the angular offset of the observer from the main beam (azimuth angle of the radiation pattern), x is the phase-center offset, r_0 is the distance from the rotational axis to the observer, and $s = r_0 - \Delta d$ is the distance from the phase center to the observer. The Δd is calculated from the change in phase, $\Delta\theta$, from the apex of the main beam to the phase at some angular offset of the radiation pattern. The difference in phase angle is converted to distance using the equation

$$\Delta d = \Delta\theta \cdot \frac{\lambda}{360} \qquad \cdot \qquad (2)$$

With r_0, φ, and Δd known, the law of cosines is used on this geometry to yield the phase-center offset from the rotational axis of the antenna pattern:

$$x_{\parallel} = \frac{-r_0(1-cos\phi) + \sqrt{r_0^2(1-cos\phi)^2 + 2\Delta d[(2r_0+\Delta d)(1-cos\phi)]}}{2(1-cos\phi)} \qquad \cdot \qquad (3)$$

The distance $x_{\parallel}$ is thus the phase-center offset component in the direction of the main beam.

2.3 Phase-Center Offset Perpendicular to Direction of the Main Beam

The second method, used to obtain the phase-center placement error's lateral to the main-beam direction, is a simple graphical method not explicitly found in the literature. It takes advantage of the fact that the lateral component of the phase-center offset will manifest itself antisymmetrically in the radiation-pattern phase relative to the main beam.

By separating the radiation-phase pattern into its symmetric and antisymmetric components about the antenna's main-beam direction, the lateral component of the offset can be calculated from the antisymmetric component. The phase difference found ±90° from the main beam of the antisymmetric portion of the radiation-pattern phase is converted to distance using Eq. 2.

Figure 5 shows the antisymmetric component of the radiation-pattern phase of the cylindrical horn antenna that has its phase-center offset 1 cm in both the main-beam and lateral direction. To obtain the lateral component of the phase-center offset, the phase excursion of the antisymmetric component, $\Delta\theta_{odd}$, is measured at an azimuth angle of ±90° from the main-beam apex. Since the component used is antisymmetric, the phase excursion can be measured at either +90°or −90°. (The algorithm developed for this report measures both the maximum and minimum excursions and averages the two.) The phase excursion is converted to distance using

$$x_\perp = \Delta\theta_{odd} \cdot \frac{\lambda}{360} \qquad \cdot \qquad (4)$$

The distance $x_\perp$ is thus the phase-center offset component's lateral to the direction of the main beam.

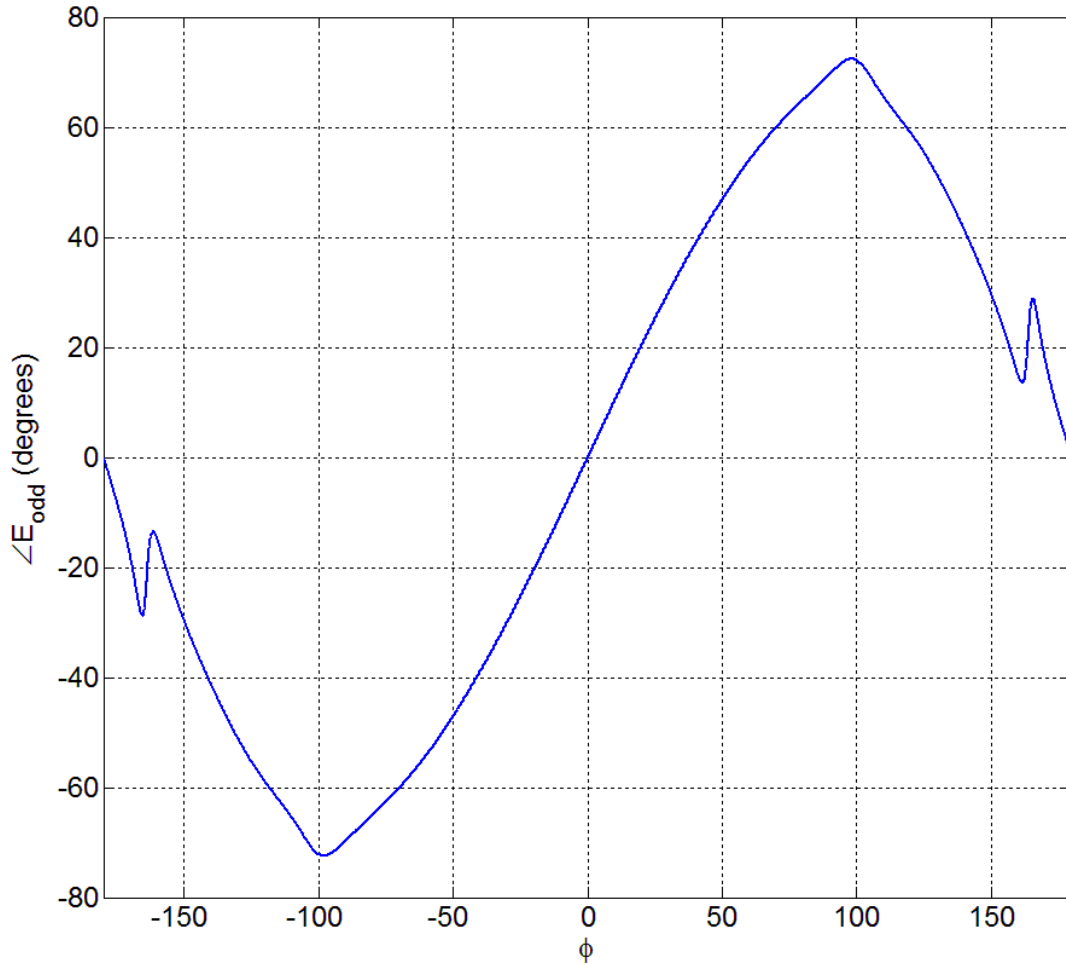## 2.4   Combining the 2 Phase-Center Offset Components

The 2 phase-center offsets from the axis of rotation are combined geometrically. Thus, in cylindrical coordinates,

$$|x| = \sqrt{x_\perp^2 + x_\parallel^2} \tag{5}$$

and

$$\angle x = arctan\frac{x_\parallel}{-x_\perp} \qquad . \tag{6}$$

3.   Analysis

To analyze the ability of the algorithm to calculate phase-center offset, the complex E-plane radiation patterns of the horn antenna with different phase-center offsets were calculated by the high-frequency structural simulator (HFSS) and put in the algorithm. To analyze the possibility of a preferential direction of numerical error, the cylindrical horn antenna was modeled with its phase-center offset –8 to 8 cm from the axis of rotation, in 1 cm increments, in both the main-beam and lateral directions. Two sets of diagonal offsets, referred to as $Diagonal_1$ and $Diagonal_2$ and having equal components of main-beam and lateral offsets, were also analyzed for error.

Figure 6 shows the error of the phase-center offset as calculated from the algorithm. Figure 6a shows the algorithm error when the cylindrical horn is displaced in the main-beam direction, while Fig. 6b shows the algorithm error when the cylindrical horn is displaced laterally to the main-beam direction. Figure 7 shows the algorithm's error when the antenna is displaced in diagonal directions from the main-beam direction (equal main-beam and lateral offset).
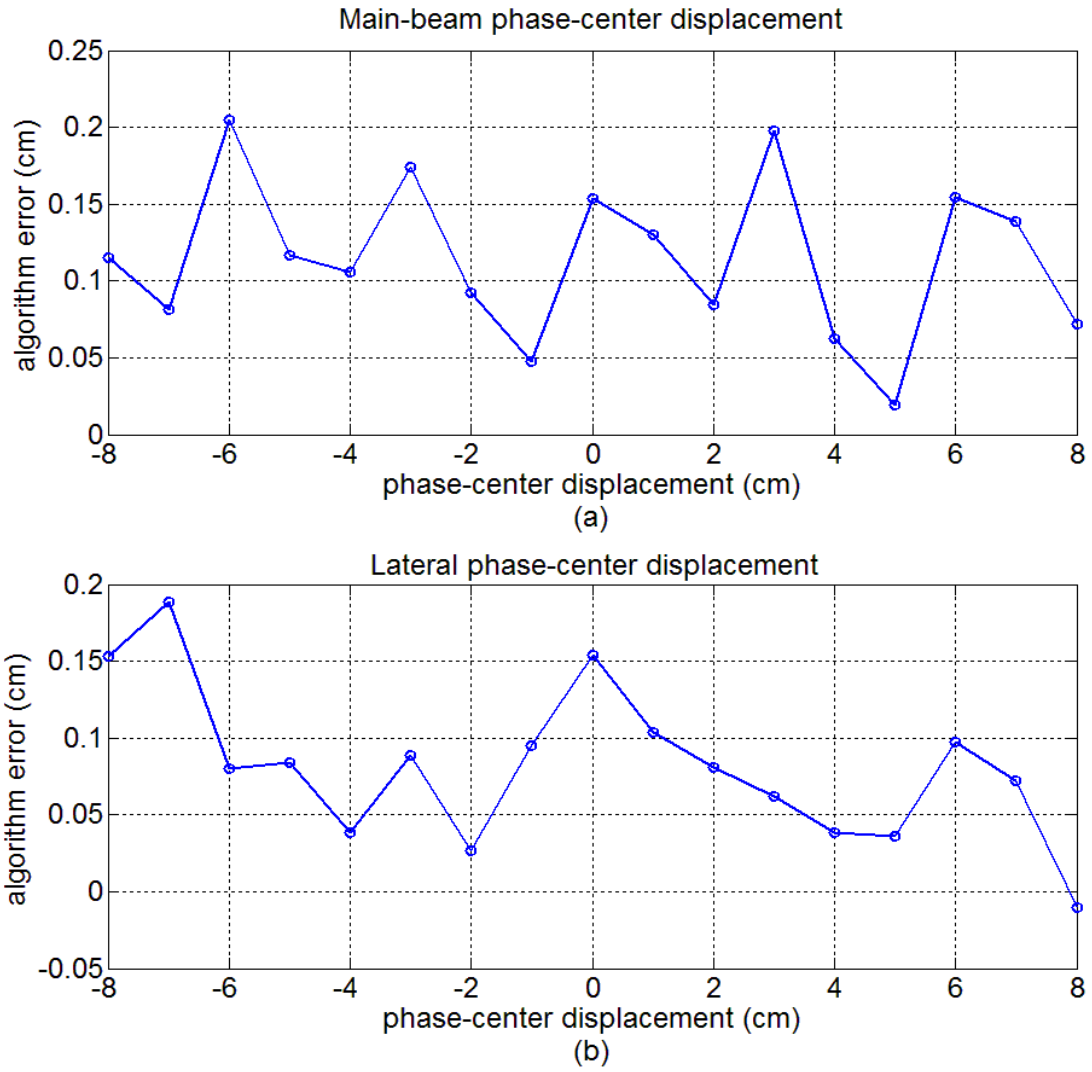
Fig. 6   Algorithm error as a function of phase-center offset: a) shows the main-beam error as the phase center is modeled offset in the main-beam direction; b) shows the lateral error when the phase center is offset in the lateral direction.
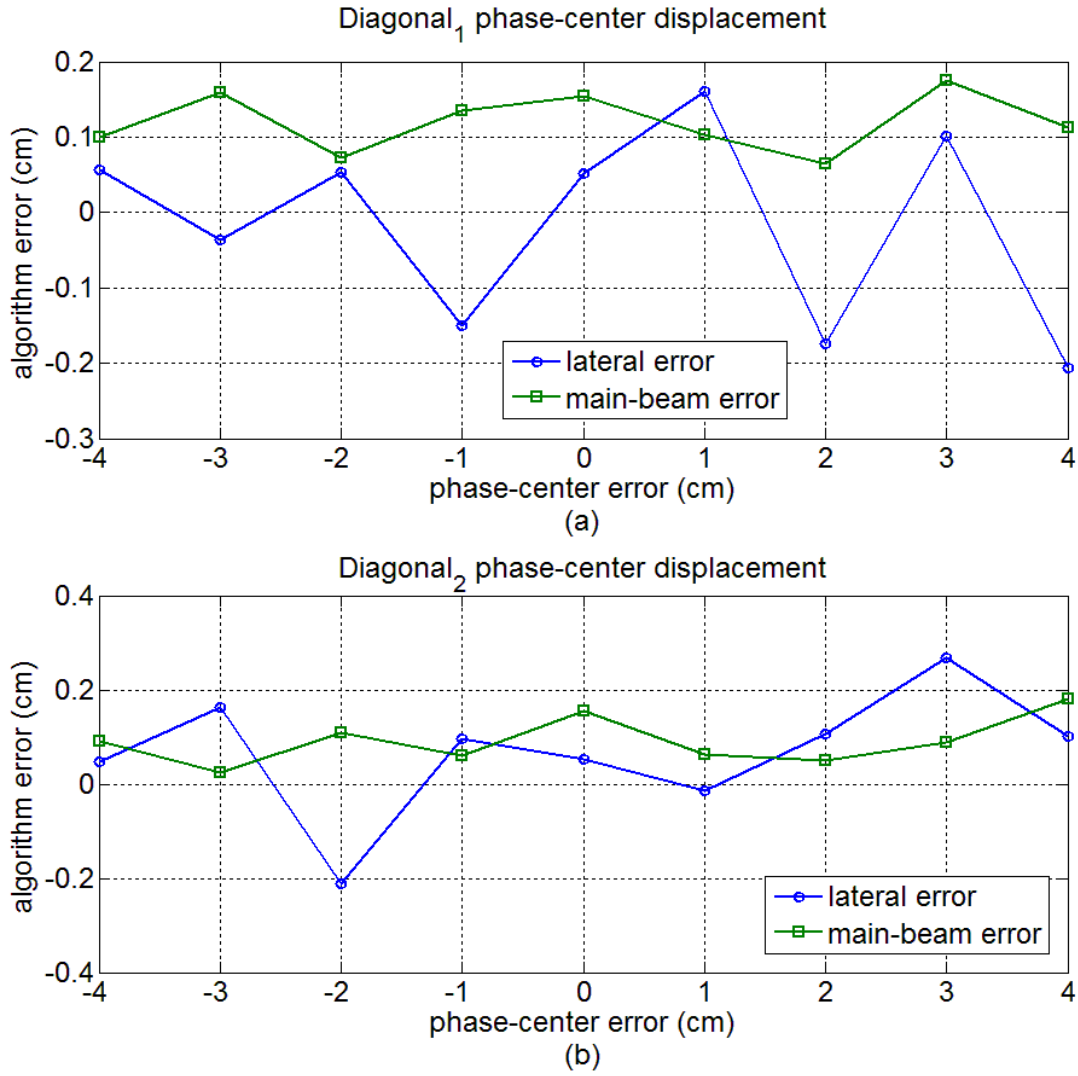
Fig. 7 Algorithm error as a function of phase-center offset: a) shows the main-beam and lateral errors as the phase center is modeled offset in the first diagonal direction; b) shows the main-beam and lateral errors when the phase center is offset in the other diagonal direction.

Figures 8 and 9 show the algorithm errors for the same offset geometry in percentage of error. The percentage of error at 0 offset was omitted due to the singularity of the calculation at that point.
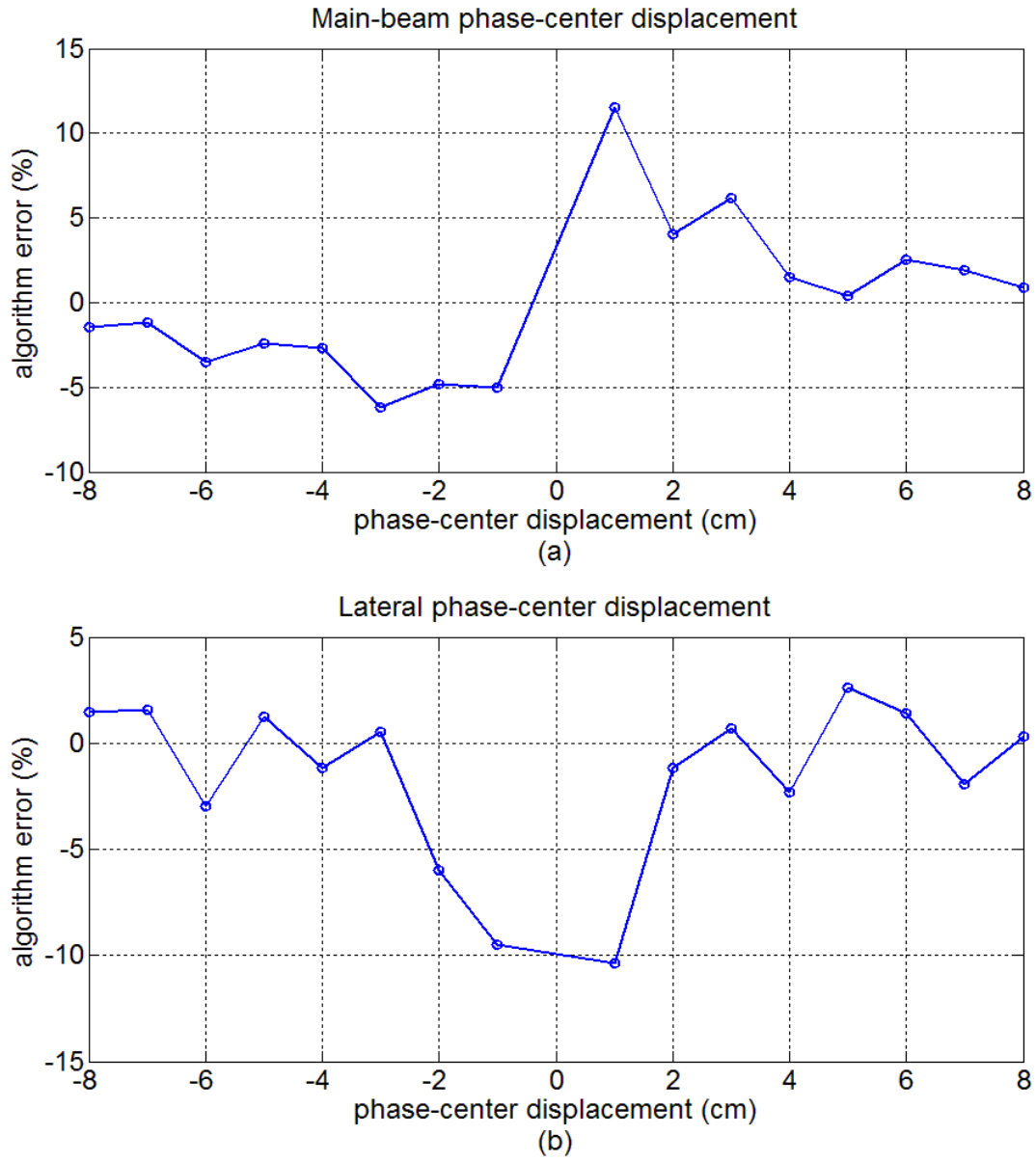
Fig. 8   Percentage of relative error as a function of phase-center offset: a) shows the main-beam error as the phase center is modeled offset in the main-beam direction; b) shows the lateral error when the phase center is offset in the lateral direction.
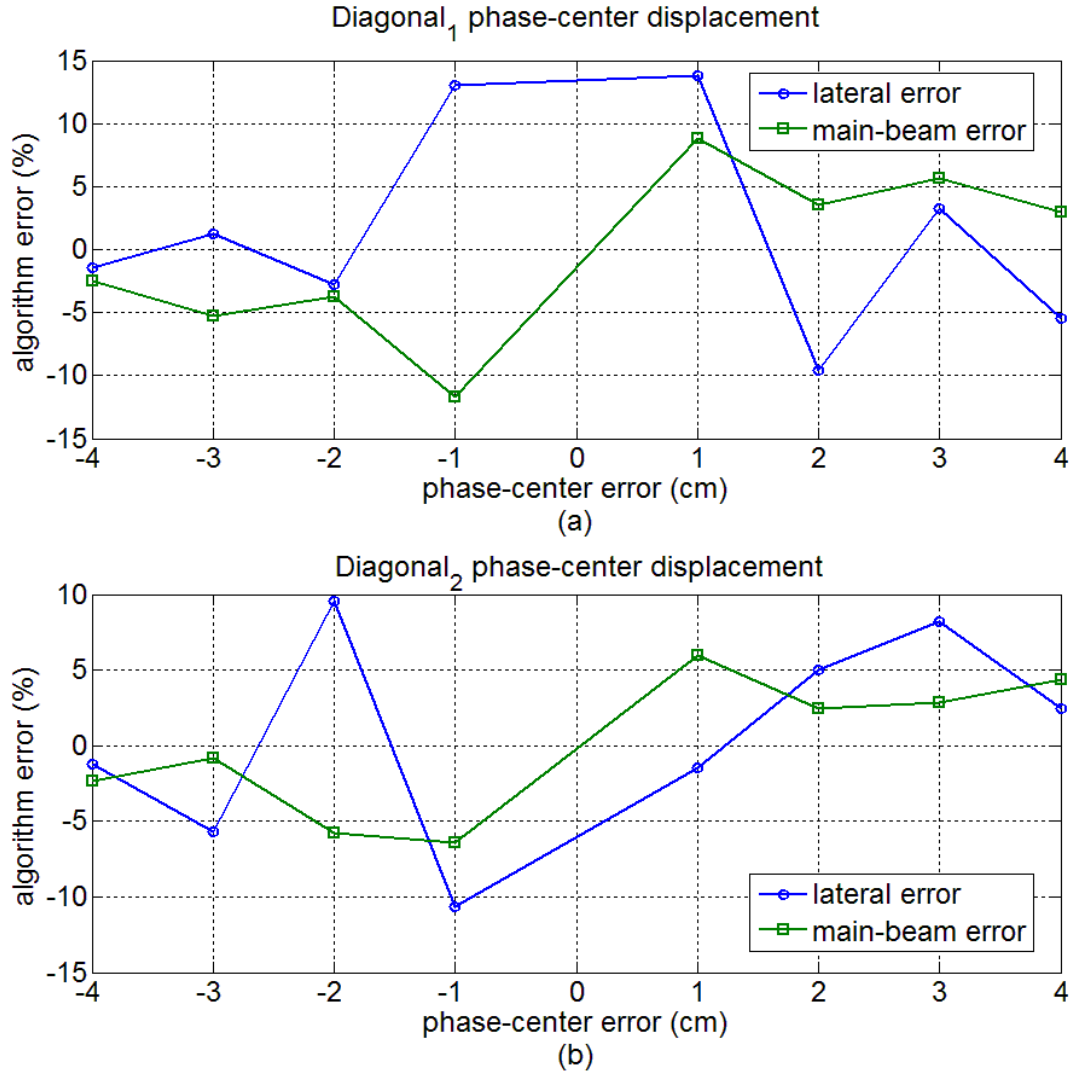
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Fig. 9   Percentage of relative error as a function of phase-center offset: a) shows the main-beam and lateral errors as the phase center is modeled offset in the first diagonal direction; b) shows the main-beam and lateral errors when the phase center is offset in the other diagonal direction.

The percentage of relative error of the algorithm is mostly within 10% in its ability to calculate the phase-center offset relative to the axis of rotation.

The operating wavelength in the modeled radiation pattern is 6 cm. The fact that no ambiguity exists in the calculations when the phase-center offset is greater than a wavelength attests to the generality of the algorithm. This is due to the fact that all phase differences are translated to distance before the calculations are performed.

4. Conclusions

An algorithm was developed that uses the complex radiation pattern to calculate the radiation phase in the far field of the antenna, and the phase-center offsets are calculated from this phase using separate methods for the main-beam direction and directions lateral to the main-beam. With the combination of the 2 phase-center-offset calculation methods, the phase-center offset's distance and direction from the rotational axis used in the radiation-pattern measurement can be calculated.

The algorithm error was analyzed by modeling the antenna with its phase-center offset the antenna's axis of rotation. The algorithm was then applied to the resulting radiation pattern, and the phase-center offsets were recovered to within 15% of their modeled values.

The offset angle used to calculate the phase-center offset in the main-beam direction is defined in the algorithm and thus can accommodate antennas with a variety of different beam widths.

Even though the phase-center location of an antenna can change as a function of frequency, a generalized algorithm has been developed to calculate the magnitude and direction of an antenna's phase-center offset from 2-dimensional radiation-pattern data at a particular frequency.

5. References

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